OPTIMAL CONTROL APPLICATIONS AND METHODS
Optim. Control Appl. Meth. 2015; **36**:844–852
Published online 8 August 2014 in Wiley Online Library (wileyonlinelibrary.com). DOI: 10.1002/oca.2138

A BMI approach to guaranteed cost control of discrete-time uncertain system with both state and input delays

Xiaojun Zhou[1,2,*,†], Tianxue Dong[1], Xiaolin Tang[1], Chunhua Yang[1] and Weihua Gui[1]

[1] *School of Information Science and Engineering, Central South University, Changsha 410083, China*
[2] *School of Science, Information Technology and Engineering, Federation University Australia, Victoria 3353, Australia*

SUMMARY

In this study, the guaranteed cost control of discrete time uncertain system with both state and input delays is considered. Sufficient conditions for the existence of a memoryless state feedback guaranteed cost control law are given in the bilinear matrix inequality form, which needs much less auxiliary matrix variables and storage space. Furthermore, the design of guaranteed cost controller is reformulated as an optimization problem with a linear objective function, bilinear, and linear matrix inequalities constraints. A nonlinear semi-definite optimization solver—PENLAB is used as a solution technique. A numerical example is given to demonstrate the effectiveness of the proposed method. Copyright © 2014 John Wiley & Sons, Ltd.

Received 3 January 2014; Revised 6 June 2014; Accepted 12 July 2014

KEY WORDS: guaranteed cost control; uncertain system; state and input delays; bilinear matrix inequality; PENLAB

1. INTRODUCTION

It is known that uncertainties and time delays may incur instability and deterioration in a system, so the robust control problem has attracted much attention in the past decades [1, 2] (readers who are interested in discrete-time uncertain delay system can refer to [3, 4] and the references therein). In real-world control problems, it is desirable not only to ensure the stability of a system but also to guarantee some level of performance. Chang and Peng [5] were the first to introduce the concept of guaranteed cost control, which can give an upper bound on a given performance index. Since then, many researchers have devoted much effort to this area [6–10].

It is shown that a wide variety of control problems can be reduced to a few standard convex optimization problems with LMI constraints, which can be solved efficiently by interior point methods [11]. By using dependent slack variables, many robust control problems can be recast into LMI formulations. However, in their treatment process, the feasible space may become much smaller or even empty, causing the corresponding results to have large conservation or make no sense. A remedy to this issue is to use the bilinear matrix inequality (BMI) formulation, which can be achieved via independent slack variables without conservation. On the other hand, some control problems, for instance, the μ/k_m synthesis for robust control design with frequency dependent scalings, are shown to be equivalent to BMI formulations [12, 13]. As a matter of fact, BMI is a more general matrix inequality than LMI, and they have this relationship: when one of the coupling variables is fixed in BMI, it will become an LMI. The first paper that introduced the concept of BMI into control theory was probably because of Safononv, Goh, and Ly [13]. Control problems via BMI formulation have gained popularity since then [7, 8, 14–16].

*Correspondence to: Xiaojun Zhou, School of Information Science and Engineering, Central South University, Changsha 410083, China.
†E-mail: tiezhongyu2010@gmail.com

In the previous work, the existence conditions of the guaranteed cost control for discrete-time uncertain delay system are mostly expressed in LMI form [6, 9, 10]. In [7], the sufficient conditions for the existence of guaranteed cost control for discrete-time uncertain system with state delay was given in the BMI form, and the numerical results showed that better least upper bound can be obtained. In this study, we consider the BMI formulation for guaranteed cost control of the discrete-time uncertain system with both state and input delays. A publicly available solver named PENLAB is used to solve the BMI problem. Experimental results have demonstrated the effectiveness of the proposed method. Compared with the LMI formulation, it is shown that the design variables (matrices) are independent and few design variables are needed in our BMI formulation; however, some of the design variables are dependent in [9], which will make the feasible space much smaller or even empty, and in [10], more design variables are introduced, which will increase the computational burden. More importantly, a better least upper bound is obtained via our BMI approach than the corresponding LMI formulation in [9, 10].

Notations: Throughout this paper, the symmetric terms in a symmetric matrix are denoted by $\begin{pmatrix} X & Y \\ * & Z \end{pmatrix} = \begin{pmatrix} X & Y \\ Y^T & Z \end{pmatrix}$.

2. PROBLEM DESCRIPTION

Let us consider the following discrete-time uncertain system with both state and input delays

$$\begin{aligned} x(k+1) = {} & (A + \Delta A(k))x(k) + (A_d + \Delta A_d(k))x(k-d) + (B + \Delta B(k))u(k) \\ & + (B_h + \Delta B_h(k))u(k-h), \end{aligned} \tag{1}$$

where $x(k) \in \mathbb{R}^n$ is the state, $u(k) \in \mathbb{R}^m$ is the control input, and d and h are the unknown constant integers representing the number of delay units in the state and input, respectively. This kind of system can be seen in many dynamical systems such as biological systems, chemical systems, and electrical networks, for example, state and input delays are very common in batch chemical processes.

It is assumed that $0 \leqslant d \leqslant d^*, 0 \leqslant h \leqslant h^*$ with d^* and h^* being known. A, A_d, B, B_h are known real-valued constant matrices with appropriate dimensions, whereas $\Delta A(k), \Delta A_d(k), \Delta B(k), \Delta B_h(k)$ are unknown real-valued matrices representing time-varying parameter uncertainties in the system model. It is assumed that the parameter uncertainties are norm-bounded with the form

$$[\Delta A(k), \Delta A_d(k), \Delta B(k), \Delta B_h(k)] = DF(k)[E_a, E_d, E_b, E_h], \tag{2}$$

where D, E_a, E_d, E_b, E_h are known real-valued constant matrices with appropriate dimensions, and $F(k)$ is an unknown matrix function satisfying

$$F^T(k)F(k) \leqslant I. \tag{3}$$

The quadratic cost functional associated with system (1) is given by

$$J = \sum_{k=0}^{+\infty} \left[x^T(k)Qx(k) + u^T(k)Ru(k) \right], \tag{4}$$

where Q and R are given symmetrical positive definite matrices with appropriate dimensions.

This paper aims to design a memoryless state feedback control law

$$u(k) = Kx(k) \tag{5}$$

for the discrete-time uncertain delay system (1), such that the resulting closed-loop system

$$x(k+1) = A_C x(k) + A_D x(k-d) + B_H K x(k-h), \tag{6}$$

DOI: 10.1002/oca

where

$$A_C = A + BK + DF(k)(E_a + E_b K),$$
$$A_D = A_d + DF(k)E_d,$$
$$B_H = B_h + DF(k)E_h,$$

is Lyapunov asymptotically stable, and the associated cost functional (4) has an upper bound, namely, $J \leqslant J^*$, where, J^* is a positive real constant.

Definition 1
Considering the discrete-time uncertain delay system (1) associated with the quadratic cost functional (4), if there exist a controller (5) $u(k) = Kx(k)$ and a positive real constant J^* such that the closed-loop system (6) is Lyapunov asymptotically stable and the value of the quadratic cost functional (4) satisfies $J \leqslant J^*$ for all admissible uncertainties, then J^* is said to be a guaranteed cost and $u(k) = Kx(k)$ is said to be a memoryless state feedback guaranteed cost control law for the system (1) with cost functional (4).

3. DESIGN OF MEMORYLESS STATE FEEDBACK GUARANTEED COST CONTROLLER

Lemma 1 (Schur complement[11]*)*
The linear matrix inequality

$$\begin{bmatrix} Q(x) & S(x) \\ S^T(x) & R(x) \end{bmatrix} > 0, \tag{7}$$

where $Q^T(x) = Q(x)$, $R^T(x) = R(x)$, and $S(x)$ depend affinely on x, is equivalent to

$$R(x) > 0, \quad Q(x) - S(x)R^{-1}(x)S^T(x) > 0, \tag{8}$$

or

$$Q(x) > 0, \quad R(x) - S^T(x)Q^{-1}(x)S(x) > 0. \tag{9}$$

Lemma 2 (Uncertainty elimination [17]*)*
Given appropriate known matrices Y, O_1, O_2 with Y symmetric and unknown matrix $F(k)$ satisfying $F^T(k)F(k) \leqslant I$, the following inequality holds

$$Y + O_1F(k)O_2 + (O_1F(k)O_2)^T < 0, \tag{10}$$

if and only if there exists a scalar $\varepsilon > 0$, such that

$$Y + \varepsilon O_1 O_1^T + \frac{1}{\varepsilon} O_2^T O_2 < 0. \tag{11}$$

Theorem 1
If there exist symmetric positive definite matrices P, S, and T with appropriate dimensions, such that the following matrix inequality holds:

$$\begin{bmatrix} \Omega & A_C^T P A_D & A_C^T P B_H K \\ * & A_D^T P A_D - S & A_D^T P B_H K \\ * & * & (B_H K)^T P B_H K - T \end{bmatrix}, \tag{12}$$

where, $\Omega = A_C^T P A_C - P + S + T + Q + K^T R K$, then the controller $u(k) = Kx(k)$ is a memoryless state feedback guaranteed cost control law for the system (1) with cost functional (4). Moreover, the value of quadratic cost functional (4) satisfies

$$J \leqslant J^* = x^T(0)Px(0) + \sum_{i=1}^{d} x^T(-i)Sx(-i) + \sum_{j=1}^{h} x^T(-j)Tx(-j). \tag{13}$$

DOI: 10.1002/oca

Proof
Construct the following Lyapunov functional candidate

$$V(k) = x^T(k)Px(k) + \sum_{i=1}^{d} x^T(k-i)Sx(k-i) + \sum_{j=1}^{h} x^T(k-j)Tx(k-j), \quad (14)$$

where P, S, and T are symmetric positive definite matrices.

The forward difference of $V(k)$ along the trajectory of the system (6) will bring

$$\begin{aligned}
\Delta V(k) &= V(k+1) - V(k) \\
&= [A_C x(k) + A_D x(k-d) + B_H K x(k-h)]^T \; P \, [A_C x(k) + A_D x(k-d) \\
&\quad + B_H K x(k-h)] - x^T(k)Px(k) + x^T(k)Sx(k) - x^T(k-d)Sx(k-d) \\
&\quad x^T(k)Tx(k) - x^T(k-h)Tx(k-h) \\
&= \eta^T(k) \begin{bmatrix} \Omega - Q - K^T R K & A_C^T P A_D & A_C^T P B_H K \\ * & A_D^T P A_D - S & A_D^T P B_H K \\ * & * & (B_H K)^T P B_H K - T \end{bmatrix} \eta(k) \\
&= \eta^T(k) \begin{bmatrix} \Omega & A_C^T P A_D & A_C^T P B_H K \\ * & A_D^T P A_D - S & A_D^T P B_H K \\ * & * & (B_H K)^T P B_H K - T \end{bmatrix} \eta(k) \\
&\quad - x^T(k)(Q + K^T R K)x(k),
\end{aligned}$$

where $\eta^T(k) = [x^T(k) \; x^T(k-d) \; x^T(k-h)]^T$.

Because matrix inequality (12) holds, we have

$$\Delta V(k) < -x^T(k)(Q + K^T R K)x(k) \leqslant 0.$$

From the Lyapunov stability theory, we can say that the closed-loop system (6) is asymptotically stable. Moreover, considering that

$$x^T(k)(Q + K^T R K)x(k) < -\Delta V(k)$$

by summing both sides of the aforementioned inequality from $k = 0$ to $+\infty$, we have

$$\begin{aligned}
J &= \sum_{k=0}^{+\infty} x^T(k)\left(Q + K^T R K\right)x(k) \leqslant V(0) - V(+\infty) \\
&= x^T(0)Px(0) + \sum_{i=1}^{d} x^T(-i)Sx(-i) + \sum_{j=1}^{h} x^T(-j)Tx(-j) = J^*.
\end{aligned}$$

□

Remark 1
It should be noted that the upper bound in (13) depends on the initial condition of system (1). In this study, it is assumed that the initial state of system (1) is arbitrary but belongs to the set

$S = \{x(-i) \in \mathbb{R}^n : x(-i) = Uv_i, v_i^T v_i \leqslant 1, i = 0, 1, 2, \cdots, \bar{d}\}$, where U is a given real-valued constant matrix and $\bar{d} = \max\{d^*, h^*\}$. As a result, the upper bound will become

$$\begin{aligned} J &\leqslant x^T(0)Px(0) + \sum_{i=1}^{d} x^T(-i)Sx(-i) + \sum_{j=1}^{h} x^T(-j)Tx(-j) \\ &= (Uv_0)^T P (Uv_0) + \sum_{i=1}^{d} (Uv_i)^T S (Uv_i) + \sum_{j=1}^{h} (Uv_j)^T T (Uv_j) \\ &\leqslant \lambda_{\max}\left(U^T P U\right) v_0^T v_0 + \sum_{i=1}^{d} \lambda_{\max}\left(U^T S U\right) v_i^T v_i + \sum_{j=1}^{h} \lambda_{\max}\left(U^T T U\right) v_j^T v_j \\ &\leqslant \lambda_{\max}\left(U^T P U\right) + d\lambda_{\max}\left(U^T S U\right) + h\lambda_{\max}\left(U^T T U\right) \\ &\leqslant \lambda_{\max}\left(U^T P U\right) + d^*\lambda_{\max}\left(U^T S U\right) + h^*\lambda_{\max}\left(U^T T U\right). \end{aligned} \tag{15}$$

Theorem 2

If there exist matrix K, symmetric positive definite matrices P, S, and T and a positive scalar ε satisfy the following BMI:

$$\begin{bmatrix} -P & P(A+BK) & PA_d & PB_hK & 0 & \varepsilon PD & 0 \\ * & -P+S+T+Q & 0 & 0 & (E_a+E_bK)^T & 0 & K^T \\ * & * & -S & 0 & E_d^T & 0 & 0 \\ * & * & * & -T & (E_hK)^T & 0 & 0 \\ * & * & * & * & -\varepsilon I & 0 & 0 \\ * & * & * & * & * & -\varepsilon I & 0 \\ * & * & * & * & * & * & -R^{-1} \end{bmatrix} < 0, \tag{16}$$

then the controller $u(k) = Kx(k)$ is a memoryless state feedback guaranteed cost control law for system (1) with cost functional (4). Moreover, the value of cost functional (4) satisfies (15).

Proof

The matrix inequality (12) can be rewritten as

$$\begin{bmatrix} A_C^T \\ A_D^T \\ (B_HK)^T \end{bmatrix} P \begin{bmatrix} A_C & A_D & B_HK \end{bmatrix} + \begin{bmatrix} -P+S+T+Q+K^TRK & 0 & 0 \\ * & -S & 0 \\ * & * & -T \end{bmatrix} < 0.$$

According to the Schur complement Lemma 1, the aforementioned inequality is equivalent to

$$\begin{bmatrix} -P^{-1} & A_C & A_D & B_HK \\ * & -P+S+T+Q+K^TRK & 0 & 0 \\ * & * & -S & 0 \\ * & * & * & -T \end{bmatrix} < 0,$$

DOI: 10.1002/oca

that is,

$$\begin{bmatrix} -P^{-1} & A+BK & A_d & B_hK \\ * & -P+S+T+Q+K^TRK & 0 & 0 \\ * & * & -S & 0 \\ * & * & * & -T \end{bmatrix} + \begin{bmatrix} 0 & DF(k)(E_a+E_bK) & DF(k)E_d & DF(k)E_hK \\ * & 0 & 0 & 0 \\ * & * & 0 & 0 \\ * & * & * & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -P^{-1} & A+BK & A_d & B_hK \\ * & -P+S+T+Q+K^TRK & 0 & 0 \\ * & * & -S & 0 \\ * & * & * & -T \end{bmatrix} + O_1F(k)O_2 + (O_1F(k)O_2)^T < 0,$$

where

$$O_1 = \begin{bmatrix} D \\ 0 \\ 0 \\ 0 \end{bmatrix}, O_2 = \begin{bmatrix} 0 & E_a + E_bK & E_d & E_hK \end{bmatrix}.$$

By the uncertainty elimination Lemma 2, the aforementioned inequality holds if and only if there exists a scalar $\varepsilon > 0$ such that

$$\begin{bmatrix} -P^{-1} & A+BK & A_d & B_hK \\ * & -P+S+T+Q+K^TRK & 0 & 0 \\ * & * & -S & 0 \\ * & * & * & -T \end{bmatrix} + \varepsilon O_1O_1^T + \frac{1}{\varepsilon}O_2^TO_2 < 0.$$

With the Schur complement Lemma 1 again, we can obtain

$$\begin{bmatrix} -P^{-1} & A+BK & A_d & B_hK & 0 & \varepsilon D & 0 \\ * & -P+S+T+Q & 0 & 0 & (E_a+E_bK)^T & 0 & K^T \\ * & * & -S & 0 & E_d^T & 0 & 0 \\ * & * & * & -T & (E_hK)^T & 0 & 0 \\ * & * & * & * & -\varepsilon I & 0 & 0 \\ * & * & * & * & * & -\varepsilon I & 0 \\ * & * & * & * & * & * & -R^{-1} \end{bmatrix} < 0.$$

Multiplying the aforementioned matrix inequality on both sides by $diag\{P, I, I, \cdots, I, I\}$, we can obtain the inequality (16). □

Remark 2

Compared with [9, 10], it can be found that the sufficient conditions expressed in the BMI form is simple and flexible, which need much less auxiliary matrix variables and storage space.

4. OPTIMIZATION PROBLEM FORMULATION AND SOLUTION TECHNIQUES

The BMI constraint (16) can be regarded as a feasibility problem in optimization. To reformulate the guaranteed cost control problem to a much more common optimization problem, by introducing additional variables α, β, γ, we can relax the upper bound (15) to

$$J^*(\alpha, \beta, \gamma) = \alpha + d^*\beta + h^*\gamma, \tag{17}$$

DOI: 10.1002/oca

where

$$U^T P U \leqslant \alpha I, \quad U^T S U \leqslant \beta I, \quad U^T T U \leqslant \gamma I. \tag{18}$$

If we get a global minimum for (17), it is obvious that the least upper bound (15) will be gained. As a result, the associated optimization problem can be formulated as

$$\begin{array}{ll} \min & J^*(\alpha, \beta, \gamma) = \alpha + d^*\beta + h^*\gamma \\ \text{s.t.} & (16), (18). \end{array} \tag{19}$$

There are several algorithms proposed for solving BMI problems, including the global and local methods. The global methods are mainly based on the brand-and-bound and branch-and-cut framework [18–21]. Existing local methods include a heuristic approach in [22], a path-following method in [23] and the popular alternative methods in [7, 8, 24], in which, it takes advantages of the fact that by fixing one of the coupling variables, the BMI problem becomes convex in terms of the remaining variables and vice versa. The same idea of alternative approach was also behind the well-known *D-K* iteration in μ/k_m synthesis [25].

To solve the LMI problem, Gahinet and Nemirovskii, *et al*. [26] wrote a software package called LMI-Lab, which was embedded into the MATLAB's LMI Control Toolbox. Unlike the LMI Toolbox, there are very few available software packages for BMI problem. That may be the main reason why LMI is much more popular than BMI. Fortunately, Kočvara [27] and his group have developed the first available general purpose BMI solver PENBMI, which is now incorporated into PENLAB [28].

In this study, we will use the PENLAB as a solution technique to solve the aforementioned optimization problem with linear objective function, bilinear and linear matrix inequalities constraints.

5. NUMERICAL RESULTS

In this section, we give a detailed example to demonstrate the effectiveness of the proposed method. Consider the discrete-time uncertain delay system (1) and the quadratic cost functional (4) with

$$A = \begin{bmatrix} 0.7 & 0 & -0.5 \\ 0.05 & 0.8 & 0 \\ 0 & 0.3 & 0.6 \end{bmatrix}, \quad A_d = \begin{bmatrix} -0.2 & 0 & 0 \\ 0 & -0.1 & 0.1 \\ 0 & 0 & -0.2 \end{bmatrix},$$

$$B = \begin{bmatrix} 0.3 \\ 0 \\ 0.6 \end{bmatrix}, \quad B_h = \begin{bmatrix} 0.1 \\ -0.3 \\ 0 \end{bmatrix}, \quad D = \begin{bmatrix} 0.1 \\ 0 \\ 0.2 \end{bmatrix},$$

$$E_a = [0.2 \quad 0 \quad 0.3], \quad E_b = 0.4, \quad E_d = 0, \quad E_h = 0.2,$$

$$Q = diag\{1, 1, 1\}, \quad R = 0.2, \quad U = diag\{1.5, 1.5, 1.5\},$$

$$d^* = 2, h^* = 1.$$

By solving the optimization problem (19) using PENLAB under MATLAB environment, we can obtain

$$K = [0.0167 \quad -0.1019 \quad -0.1594], \quad \varepsilon = 0.0481,$$

$$P = \begin{bmatrix} 11.7109 & -0.7037 & -11.8333 \\ -0.7037 & 79.8606 & 8.1939 \\ -11.8333 & 8.1939 & 41.9930 \end{bmatrix}, \quad S = \begin{bmatrix} 2.3128 & -0.8976 & -2.2887 \\ -0.8976 & 9.7071 & -3.8349 \\ -2.2887 & -3.8349 & 7.3816 \end{bmatrix},$$

$$T = \begin{bmatrix} 0.0291 & -0.1784 & -0.2789 \\ -0.1784 & 1.0915 & 1.7066 \\ -0.2789 & 1.7066 & 2.6684 \end{bmatrix},$$

$$\alpha = 183.8258, \quad \beta = 28.3382, \quad \gamma = 8.5253.$$

DOI: 10.1002/oca

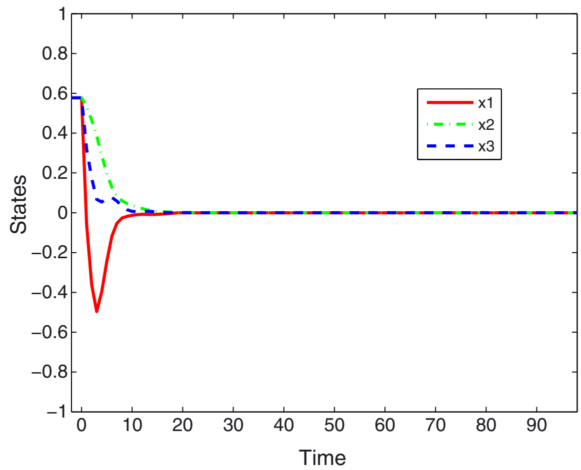

Figure 1. State variables changing of the system via the optimal guaranteed cost controller.

By Theorem 2, the optimal guaranteed cost controller is

$$u(k) = [0.0167 \quad -0.1019 \quad -0.1594]x(k)$$

and the least upper bound $J^* = 249.0275$. Compared with $J^* = 568.6914$ in [9] and $J^* = 252.9535$ in [10], we can find that Theorem 2 gives a less conservative result than those in [9, 10].

Moreover, for specification, let $d = 2, h = 1, F(k) = \sin(k), x(-2) = x(-1) = x(0) = [0.5774, 0.5774, 0.5774]^T$, Figure 1 shows that the system can be well stabilized via the optimal guaranteed cost controller.

Remark 3

The numerical experiment shows that for a similar Lyapunov functional candidate in [9], the BMI formulation can get much better result, whereas for a different Lyapunov functional candidate in [10], the result obtained by our BMI approach is better as well.

6. CONCLUSION

In this study, we have considered the guaranteed cost control of discrete-time uncertain system with both state and input delays. We have given a BMI formulation for the existence conditions of memoryless state feedback guaranteed cost controller. A publicly available solver is used to solve the BMI problem globally, and the experimental results have shown the advantages of the proposed method.

It should be noted that the guaranteed cost control problem can be considered as a bilevel optimization problem. In the first level, a Lyapunov functional candidate is chosen to guarantee the stability or feasibility, whereas in the second level, a quadratic performance index is given to be optimized. Actually, there exist two NP-hard problems in this issue, one is how to choose an appropriate Lyapunov functional candidate and the second is how to optimize an equivalent BMI problem. On the other hand, the bilevel optimization problem can be combined into one, namely, to find an optimal state feedback gain such that the quadratic cost functional is minimized, because if the optimal solution is achieved, the state must be stable, which indicates potential ways in our future work.

DOI: 10.1002/oca

ACKNOWLEDGEMENTS

The work was supported by the National Science Found for Distinguished Young Scholars of China (Grant No. 61025015), the Foundation for Innovative Research Groups of the National Natural Science Foundation of China (Grant No. 61321003), and the China Scholarship Council.

REFERENCES

1. Lin C, Wang QG, Lee TH. A less conservative robust stability test for linear uncertain time-delay systems. *IEEE Transactions on Automatic Control* 2006; **51**(1):87–91.
2. Moon YS, Park P, Kwon WH, Lee YS. Delay-dependent robust stabilization of uncertain state-delayed systems. *International Journal of Control* 2001; **74**(14):1447–1455.
3. Gao H, Chen TW. New results on stability of discrete-time systems with time-varying state delay. *IEEE Transactions on Automatic Control* 2007; **52**(2):328–334.
4. He Y, Wu M, Liu GP, She JH. Output feedback stabilization for a discrete-time system with a time-varying delay. *IEEE Transactions on Automatic Control* 2008; **53**(10):2372–2377.
5. Chang SSSL, Peng T. Adaptive guaranteed cost control of systems with uncertain parameters. *IEEE Transactions on Automatic Control* 1972; **17**(4):474–483.
6. Chen WH, Guan ZH, Lu XM. Delay-dependent guaranteed cost control for uncertain discrete-time systems with both state and input delays. *Journal of the Franklin Institute* 2004; **341**(5):419–430.
7. Nian XH, Sun ZM, Wang HB, Zhang H, Wang X. Bilinear matrix inequality approaches to robust guaranteed cost control for uncertain discrete-time delay system. *Optimal Control Applications and Methods* 2012; **34**(4):433–441.
8. Nian XH, Sun ZM, Wang X. Robust guaranteed cost decentralized stabilization for uncertain discrete large-scale systems with delays via state feedback and output feedback. *Journal of Optimization Theory and Applications* 2012; **155**:694–706.
9. Yu L, Gao FR. Optimal guaranteed cost control of discrete-time uncertain systems with both state and input delays. *Journal of the Franklin Institute* 2001; **338**(1):101–110.
10. Zuo ZQ, Wang YJ. Novel optimal guaranteed cost control of uncertain discrete systems with both state and input delays. *Journal of Optimization Theory and Applications* 2008; **139**(1):159–170.
11. Boyd S, El Ghaoui L, Feron E, Balakrishnan V. *Linear Matrix Inequalities in System and Control Theory*. SIAM: Philadelphia, PA, 1994.
12. Goh KC, Safonov MG, Ly JH. Robust synthesis via bilinear matrix inequalities. *International Journal of Robust and Nonlinear Control* 1996; **6**(9):1079–1095.
13. Safonov MG, Goh KC, Ly JH. Control system synthesis via bilinear matrix inequalities. *Procedings of the American Control Conference*, Vol. 1: Baltimore, MD, June 1994; 45–49.
14. Kanev S, Scherer C, Verhaegen M, De Schutter B. Robust output-feedback controller design via local BMI optimization. *Automatica* 2004; **40**(7):1115–1127.
15. VanAntwerp JG, Braatz RD, Sahinidis NV. Globally optimal robust control for systems with time-varying nonlinear perturbations. *Computers & Chemical Engineering* 1997; **21**:125–130.
16. VanAntwerp JG, Braatz RD. A tutorial on linear and bilinear matrix inequalities. *Journal of Process Control* 2000; **10**(4):363–385.
17. Petersen IR. A stabilization algorithm for a class of uncertain linear systems. *Systems & Control Letters* 1987; **8**(4):351–357.
18. Beran E, Vandenberghe L, Boyd S. A global BMI algorithm based on the generalized Benders decomposition. *Proceedings of the European Control Conference*, Brussels, Belgium, 1997; 1074–1082.
19. Fukuda M, Kojima M. Branch-and-cut algorithms for the bilinear matrix inequality eigenvalue problem. *Computational Optimization and Applications* 2001; **19**(1):79–105.
20. Goh KC, Safonov MG, Papavassilopoulos GP. Global optimization for the biaffine matrix inequality problem. *Journal of Global Optimization* 1995; **7**(4):365–380.
21. Tuan DH, Apkarian P, Nakashima Y. A new Lagrangian dual global optimization algorithm for solving bilinear matrix inequalities. *Proceedings of the American Control Conference*, Vol. 3: San Diego, California, June 1999; 1851–1855.
22. Zheng F, Wang QG, Lee TH. A heuristic approach to solving a class of bilinear matrix inequality problems. *Systems & Control Letters* 2002; **47**(2):111–119.
23. Hassibi A, How J, Boyd S. A path-following method for solving BMI problems in control. *Proceedings of the American Control Conference*, Vol. 2: San Diego, California, June 1999; 1385–1389.
24. Iwasaki I. The dual iteration for fixed-order control. *IEEE Transactions on Automatic Control* 1999; **44**(4):783–788.
25. Doyle JC. Synthesis of robust controllers and filters. *IEEE Conference on Decision and Control* 1983; **22**:109–114.
26. Gahinet P, Nemirovskii A, Laub AJ, Chilali M. The LMI control toolbox. *IEEE Conference on Decision and Control* 1994; **3**:2038–2041.
27. Henrion D, Löfberg J, Kocvara M, Stingl M. Solving polynomial static output feedback problems with PENBMI. *IEEE Conference on Decision and Control* 2005:7581–7586.
28. Fiala J, Kočvara M, Stingl M. PENLAB: a MATLAB solver for nonlinear semidefinite optimization. (Available from: http://arxiv.org/abs/1311.5240) [Accessed on 20 November 2013].

DOI: 10.1002/oca